Mass generation for gauge fields in the Salam-Weinberg theory without Higgs

J. Barcelos-Neto* and S. Rabello†
Instituto de Física
Universidade Federal do Rio de Janeiro
RJ 21945-970 - Caixa Postal 68528 - Brasil

Abstract

We consider the Salam-Weinberg theory by introducing tensor gauge fields. When these fields are coupled in a topological way with the vector ones, the resulting system constitutes an alternative mechanism of mass generation for vector fields without the presence of Higgs bosons. We show that these masses are in agreement with the ones obtained by means of the spontaneous symmetry breaking.

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*e-mail: barcelos@if.ufrj.br
†e-mail: rabello@if.ufrj.br - Address after February 19th, 1996: Department of Physics, Stanford University, Stanford, CA94305-4060, USA

1. The origin of mass generation for gauge fields in the Salam-Weinberg (SW) theory has been an interesting and intriguing problem since its proposal. Nowadays, it is widely accepted that spontaneous symmetry breaking together with the Higgs mechanism is the most probable explanation for the origin of these masses. However, if this is actually true, the Higgs bosons must exist in nature. The point is that there is no precise theoretical prediction on the mass scale where these fields could be found and experiments till now have shown no evidence about them.

More recently, it has been pointed out that a vector-tensor gauge theory [1], where vector and tensor fields are coupled in a topological way by a kind of a Chern-Simons term, constitutes an interesting mechanism of mass generation for vector fields that is not plagued with Higgs. The general idea of this mechanism resides in the following: Tensor gauge fields [2] are antisymmetric quantities and consequently in $D = 4$ they exhibit six degrees of freedom. By virtue of the massless condition, the number of degrees of freedom goes down to four. Since the gauge parameter is a vector quantity, this number would be zero if all of its components were independent. This is nonetheless the case because the system is reducible and we mention that the final number of physical degrees of freedom is one. When the Chern-Simons term is introduced, where vector and tensor field are coupled in a topological way, this remaining tensor degree of freedom can be absorbed by the vector one to acquire mass [1, 3]. We mention that this peculiar structure of constraints implies that quantization deserves some care and a reasonable amount of work has been done on this subject [4].

The purpose of our paper is to use this mechanism in the SW theory in order to generate mass for the weak gauge fields.

2. Let us first briefly show how these ideas work out in the Abelian case. We start from the well-known Lagrangian of the Maxwell electromagnetic theory

$$\mathcal{L} = -\,\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu}\,, \tag{1}$$

where the tensor field $F_{\mu\nu}$ is defined as usual

$$F_{\mu\nu} = \partial_\mu A_\nu - \partial_\nu A_\mu\,. \tag{2}$$

Let us now suppose that we would like to have massive photons. If we directly put a mass term into the Lagrangian like

$$\mathcal{L} = -\,\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} + \frac{1}{2}\,m^2 A_\mu A^\mu\,, \tag{3}$$

we would have two problems (maybe more): The theory would lose the gauge invariance and would not be renormalizable any more. Even with these problems let us rewrite the Lagrangian (3) with the help of an auxiliary field as follows

$$\mathcal{L} = -\,\frac{1}{4}\,F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}\,j_\mu j^\mu + m\,j_\mu A^\mu\,. \tag{4}$$

We observe that the calculation of the equation of motion for j_μ and its replacement back into (4) leads to the previous Lagrangian (3).

The important part of the present development is to look at the Lagrangian (4) again but without considering it necessarily related to (3). This occurs when we take j_μ as a function of another field. In this case, we cannot assume that (3) and (4) are equivalent anymore, even classically. The interesting point is that the gauge invariance, lost in (3), can be restored in the Lagrangian (4) if we assume that j_μ exhibits the following properties: off-shell divergenceless and gauge invariance. It is necessary to be off-shell divergenceless in order to compensate the gauge transformation of A_μ, i.e.

$$\begin{aligned} \delta A_\mu\, j^\mu &= \partial_\mu \alpha\, j^\mu\,, \\ &\longrightarrow -\alpha\, \partial_\mu j^\mu\,, \\ &= 0\,, \end{aligned} \tag{5}$$

where the second step above contains a total derivative. Concerning the gauge invariance of j_μ, it is an assumption that can always be done, in principle for the Abelian case.

In order to fulfill these two conditions, tensor fields emerge naturally by writing j_μ as a topological quantity [1]

$$j_\mu = \frac{1}{2}\, \epsilon_{\mu\nu\rho\lambda}\, \partial^\nu B^{\rho\lambda}\,. \tag{6}$$

We assume that $B^{\mu\nu}$ is independent of the gauge transformation of the vector field, characterized by the parameter $\alpha(x)$ above. Consequently, the gauge invariance condition for the topological current j_μ is directly verified. On the other hand, the antisymmetric tensor field can have its own gauge transformation. Using the vector parameter $\xi_\mu(x)$ to characterize it, we have

$$\delta B^{\rho\lambda}(x) = \partial^\rho \xi^\lambda(x) - \partial^\lambda \xi^\rho(x)\,. \tag{7}$$

The gauge transformations given by (7) are not all independent. We notice that $\delta B^{\rho\lambda} = 0$ if ξ^ρ is replaced by the derivative of some scalar quantity. We also verify that j_μ remains invariant for the gauge transformation given by (7). If we assume that A_μ does not depend on it, the Lagrangian (4) will be invariant for these two gauge transformations.

Summarizing all the results above, we have

[1] We use the convention that the scalar product between two antisymmetric quantities shall display a factor that takes care of the multiplicity of terms. This is the reason for the factor 1/2 in expression (6). We are also considering that $\epsilon^{0123} = 1 = -\,\epsilon_{0123}$ and that the flat metric tensor reads $\eta_{\mu\nu} = (+,-,-,-) = \eta^{\mu\nu}$.

$$
\begin{array}{ll}
(i) & \text{Vector gauge transformations :} \\
& \delta_\alpha A_\mu = \partial_\mu \alpha \,, \\
& \delta_\alpha B_{\mu\nu} = 0 \,, \\
& \delta_\alpha j_\mu = 0 \,,
\end{array} \tag{8}
$$

$$
\begin{array}{ll}
(ii) & \text{Tensor gauge transformations :} \\
& \delta_\xi B_{\mu\nu} = \partial_\mu \xi_\nu - \partial_\nu \xi_\mu \,, \\
& \delta_\xi A_\mu = 0 \,, \\
& \delta_\xi j_\mu = 0 \,,
\end{array} \tag{9}
$$

We have used different subscripts to denote both transformations. Making now the replacement of j_μ, given by (6), into the Lagrangian (4), we obtain

$$
\begin{aligned}
-\frac{1}{2} j_\mu j^\mu &= -\frac{1}{8} \epsilon_{\mu\nu\rho\lambda} \epsilon^{\mu\zeta\alpha\beta} \partial^\nu B^{\rho\lambda} \partial_\zeta B_{\alpha\beta} \,, \\
&\equiv -\frac{1}{72} \epsilon_{\mu\nu\rho\lambda} \epsilon^{\mu\zeta\alpha\beta} H^{\nu\rho\lambda} H_{\zeta\alpha\beta} \,, \\
&= \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} \,,
\end{aligned} \tag{10}
$$

where the tensor $H_{\mu\nu\rho}$ is defined as

$$
H_{\mu\nu\rho} = \partial_\mu B_{\nu\rho} + \partial_\rho B_{\mu\nu} + \partial_\nu B_{\rho\mu} \,. \tag{11}
$$

We write down the final expression of the Lagrangian as it usually appears in literature [1, 2, 3, 4]

$$
\mathcal{L} = -\frac{1}{4} F_{\mu\nu} F^{\mu\nu} + \frac{1}{12} H_{\mu\nu\rho} H^{\mu\nu\rho} + \frac{1}{2} m \, \epsilon_{\mu\nu\rho\lambda} A^\mu \partial^\nu B^{\rho\lambda} \,. \tag{12}
$$

It is important to emphasize that the Lagrangian above, although gauge invariant, effectively describes a massive vector gauge field [1]. This is achieved, for example, by considering the path integral formalism and integrating out the tensor fields. An effective Lagrangian for vector fields is then obtained. Their propagators present a massive pole with the same mass m of the classical analysis given by the combination between (3) and (4). For details, see reference [3]. It might also be opportune to mention that the mass generation embodied in (12) is symmetrical, that is to say, the elimination of vector field gives also mass to the tensor one.

3. In order to implement these ideas in the SW, it is necessary to have the non-Abelian formulation of the vector-tensor gauge theory. Let us mention that

this is not a trivial subject [5, 6] and we can directly understand why this occurs. The non-Abelian version of the tensor gauge transformation (7) must be

$$\delta B^a_{\mu\nu} = (D_\mu \xi_\nu)^a - (D_\nu \xi_\mu)^a \,. \tag{13}$$

Here we notice that if we replace the gauge parameter ξ^a_μ by a derivative (even covariant) of some spacetime scalar we do not get zero as in the Abelian case. So, the reducibility condition does not happen in the non-Abelian formulation. This is the origin of the problem. A non-Abelian gauge theory is incompatible with the Abelian limit because there is a discontinuity between the two sectors (the Abelian case has more symmetries than the non-Abelian one). A possible solution for this problem is to introduce a kind of Stuckelberg field in order to make compatible the symmetries of the two sectors [6]. However, for our particular purposes in the present paper, that is just to calculate the masses of free vector fields, we do not need to know details of higher order interaction involving vector and tensor fields. These masses are obtained as being poles of the propagators of the vector fields.

Let us then write down the gauge field sector of the SW theory

$$\mathcal{L}_g = -\frac{1}{4}\, F^a_{\mu\nu} F^{a\,\mu\nu} - \frac{1}{4}\, F_{\mu\nu} F^{\mu\nu} \,, \tag{14}$$

where

$$\begin{aligned} F^a_{\mu\nu} &= \partial_\mu W^a_\nu - \partial_\nu W^a_\mu + g\,\epsilon^{abc}\, W^b_\mu\, W^c_\nu \,, & (15)\\ F_{\mu\nu} &= \partial_\mu B_\nu - \partial_\nu B_\mu \,. & (16) \end{aligned}$$

Here, W^a_μ are the gauge fields related to the $SU(2)_L$ symmetry and B_μ to the $U(1)$ hypercharge. This last one is a combination between electromagnetic and neutral weak fields. The same occurs with W^3_μ. These combinations are expressed in terms of the Weinberg angle θ_W as follows

$$\begin{aligned} B_\mu &= \cos\theta_W\, A_\mu - \sin\theta_W\, Z_\mu \,, \\ W^3_\mu &= \sin\theta_W\, A_\mu + \cos\theta_W\, Z_\mu \,. \end{aligned} \tag{17}$$

In order to obtain mass for gauge fields, we follow the same procedure of the Abelian case and introduce the Lagrangian

$$\mathcal{L}_j = -\frac{1}{2} j^a_\mu\, j^{a\mu} - \frac{1}{2} j_\mu\, j^\mu + M\, j^a_\mu\, W^{a\mu} + M'\, j_\mu\, B^\mu + \dots \,, \tag{18}$$

where dots are representing the remaining terms related to the non-Abelian formulation [5, 6]. Classically, if one eliminates j_μ and j^a_μ by using their equation of motion, the mass terms $\frac{1}{2} M W_{a\mu} W^{a\mu}$ and $\frac{1}{2} M' B_\mu B^\mu$ will be obtained. the same

occurs in the quantum point of view, when tensor gauge fields are introduced [3]. Since the mass poles obtained in the quantum propagators are the same of the classical formalism, when tensor gauge fields are eliminated, we continue to work classically throughout the paper. This avoid us to run into desnecessary algebraic complications.

Of course, we do not want a mass generation like that, where the photon field also becomes massive. Let us then use the combination given by (17) into the last two terms of (18). The result is

$$\begin{aligned} M\, j^3_\mu\, W^{3\mu} + M'\, j_\mu\, B^\mu &= (M \sin\theta_W\, j^{3\mu} + M' \cos\theta_W\, j^\mu)\, A_\mu \\ &+ (M\cos\theta_W\, j^{3\mu} - M'\sin\theta_W\, j^\mu)\, Z_\mu\,. \end{aligned} \tag{19}$$

Since we do not want a mass generation for the photon field, we have that j_μ and j^3_μ cannot be independent. We thus take

$$M \sin\theta_W\, j^{3\mu} + M' \cos\theta_W\, j^\mu = 0\,. \tag{20}$$

This permit us to also eliminate the topological current j^μ. Hence,

$$\begin{aligned} &-\frac{1}{2} j^3_\mu\, j^{3\mu} - \frac{1}{2} j_\mu\, j^\mu + M\, j^3_\mu\, W^{3\mu} + M'\, j_\mu\, B^\mu \\ &\qquad = -\frac{1}{2}\left[1 + \left(\frac{M}{M'}\right)^2 \tan^2\theta_W\right] j^3_\mu\, j^{3\mu} + \frac{M}{\cos\theta_W}\, j^3_\mu\, Z^\mu \end{aligned} \tag{21}$$

The equation for j^3_μ then reads

$$j^3_\mu = \frac{\dfrac{M}{\cos\theta_W}}{1 + \left(\dfrac{M}{M'}\right)^2 \tan^2\theta_W}\, Z_\mu \tag{22}$$

Replacing it back into (21), we get

$$\begin{aligned} &-\frac{1}{2} j^3_\mu\, j^{3\mu} - \frac{1}{2} j_\mu\, j^\mu + M\, j^3_\mu\, W^{3\mu} + M'\, j_\mu\, B^\mu \\ &\qquad = -\frac{1}{2}\, \frac{\left(\dfrac{M}{\cos\theta_W}\right)^2}{1 + \left(\dfrac{M}{M'}\right)^2 \tan^2\theta_W}\, Z_\mu Z^\mu \end{aligned} \tag{23}$$

We thus observe that the mass generated for Z_μ reads

$$M_Z = \frac{M}{\cos\theta_W \sqrt{1 + \left(\frac{M}{M'}\right)^2 \tan^2\theta_W}} \tag{24}$$

where M is the mass of W^1_μ and W^2_μ. Here, M' is a free parameter that has to be conveniently fixed, accordingly the experiment. Since we already know that M and M_Z are related by $M_Z = M/\cos\theta_W$, we conclude that M' must be an infinite parameter (or $M' \gg M$). Our procedure is then in agreement with the results of the spontaneous symmetry breaking of the SW theory. But we emphasize that there are no Higgs bosons here.

4. In conclusion, we have used an alternative mechanism of mass generation for gauge fields in the SW theory, without Higgs, that is based on a vector-tensor gauge theory where vector and tensor fields are coupled in a topological way. The physical interpretation of our result is that the massive vector fields, that we effectively see in nature, must be related to massless vector and tensor gauge fields at some stage of the same nature. We would like to stress that this might not be an isolated fact, just considered to be an alternative mass generation mechanism. Antisymmetric tensor degrees of freedom might be the reason for the intriguing spacetime dimension $D = 10$ of superstring theories. There is a possibility of anomaly cancellation in these theories for $D = 4$ if antisymmetric tensor degrees of freedom are introduced [7].

Unfortunately, there remains an open question. The present mechanism does not appear to be appropriate to generate mass for matter fields. We guess that this point requires a better comprehension of the role played by the masses of the matter fields in the context of the SW theory. This point is presently under study and possible results shall be reported elsewhere [8]

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